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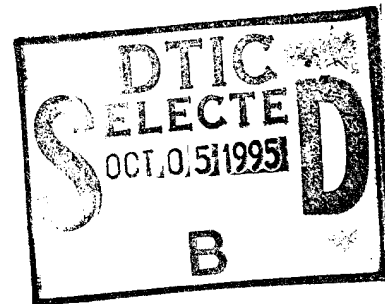
TRYPHON T. GEORGIU  
JULY 1995

FINAL REPORT

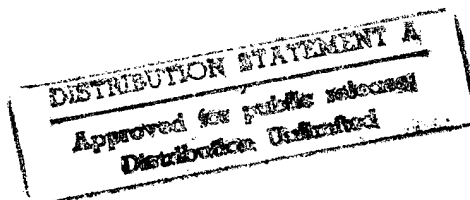
ASPECTS OF MODELING, IDENTIFICATION AND  
CONTROL OF DYNAMICAL SYSTEMS

Submitted to the  
Air Force Office of Scientific Research  
Directorate of Mathematical  
and Computer Sciences

AFOSR Grant No. AF/F49620-92-J-0241



Department of Electrical Engineering  
University of Minnesota  
Minneapolis, MN 55455



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## Summary

During the period May 1, 1992 to April 30, 1995, research, supported by the Air Force Office for Scientific Research under Grant No. AF/F49620-92-J-0241, was carried out to develop certain analysis and design tools for an integrated approach to modeling, identification, and control of dynamical systems.

The research developed an input-output framework based on the notion of *graph* and the *gap metric*. The notion of graph allows a representation independent characterization of systems. The gap metric is a measure of distance between dynamical systems which is used to quantify modeling uncertainty. This measure is especially natural because it captures the "coarsest" type of uncertainties that feedback systems tolerate.

The research supported by the grant developed an approach for robust control system design and a compatible theory for identification and approximation of dynamical systems. Initially it dealt with the case of linear systems (lumped as well as distributed parameter systems). Case studies highlighted the use of the techniques, and computer software was prepared, which is available as part a control design toolbox (Matlab's  $\mu$ -tools toolbox). In the last year of the grant the research advanced an analogous framework for robust control and analysis of nonlinear systems. In particular, robustness measures were developed and a methodology for computing induced norms was obtained. The supported research also focused on implementation issues of sampled-data control systems, the effects of persistent excitations in control systems, and numerical aspects of spectral factorization.

The research supported by the present grant resulted in 16 journal papers (3 papers under review, 1 in preparation), 17 refereed conference publications, 2 book chapters, and two Ph.D. theses (one in the final stages of completion).

# 1 Research Publications

The research supported by the present grant resulted in 16 journal papers (3 under review, 1 in preparation), 17 refereed conference publications, and 2 book chapters. These are listed below.

1. T.T. Georgiou and M.C. Smith, "Robust stabilization in the gap metric: Controller design for distributed plants", *IEEE Trans. on Automatic Control*, **37**: 1133-1143, August 1992.
2. T.T. Georgiou and M.C. Smith, "Upper and lower bounds for approximation in the gap metric," *IEEE Trans. on Automatic Control*, **38**(6): 946-951, 1993.
3. S.Buddie, T.T. Georgiou, Ümit Özgüner, and M.C. Smith, "Flexible Structure Experiments at JPL and WPAFB", *International Journal of Control*, **58**(1): 1-19, 1993.
4. J.C. Doyle, T.T. Georgiou, and M.C. Smith, "The parallel projection operators of a nonlinear feedback system," *Systems and Control letters*, **20**: 79-85, 1993.
5. C. Shankwitz and T.T. Georgiou, "On the envelope of spectral power of stochastic processes," *IEEE Trans. on Information Theory*, **40**(2): 584-588, 1994.
6. T.T. Georgiou, "Differential stability and robust control of nonlinear systems," *Mathematics of Control, Signals and Systems*, **6**(4): 289-307, 1993.
7. T.T. Georgiou and M.C. Smith, "Graphs, causality and stabilizability: linear, shift-invariant systems on  $L_2[0, \infty)$ ," *Mathematics of Control, Signals and Systems*, **6**(3): 195-223, 1993.
8. C.Chang and T.T. Georgiou, "Geometric aspects of the Caratheodory extension problem," *Linear Algebra and its Applications*, **203/4**: 209-251, 1994.
9. C.Chang and T.T. Georgiou, "Spectral factorization and Nevanlinna-Pick interpolation: State-space formulae", *IEEE Trans. on Circuits and Systems*, under revision.
10. I. Fialho and T.T. Georgiou, "On Stability and performance of sampled-data systems subject to wordlength constraint" *IEEE Trans. on Automatic Contr.* , **39**(12): 2476-2481, December 1994,
11. H. Dym, T.T. Georgiou, and M.C. Smith, "Explicit formulas for optimally robust controllers for delay systems," *IEEE Trans. on Automatic Contr.* , **40**(4): 656-669, April 1995.
12. T.T. Georgiou and M.C. Smith, "Intrinsic difficulties in using the doubly-infinite time axis for input-output systems theory," *IEEE Trans. on Automatic Contr.* , **40**(3): 516-518, March 1995.

13. I. Fialho and T.T. Georgiou, "On the  $\mathcal{L}_1$ -norm of uncertain linear systems," *IEEE Trans. on Automatic Contr.* , **40**(6): 1142-1147, June 1995.
14. I. Fialho and T.T. Georgiou, " $\ell_1$  state-feedback control with a prescribed rate of exponential convergence," *IEEE Trans. on Automatic Contr.* , submitted for publication, May 1995.
15. I. Fialho and T.T. Georgiou, " $\mathcal{L}_\infty$ -gain analysis of nonlinear systems," *IEEE Trans. on Automatic Contr.* , submitted for publication, May 1995.
16. T.T. Georgiou and M.C. Smith, "Distance Measures for Uncertain Nonlinear Systems," report in preparation for journal submission.

## Book Chapters

17. T. T. Georgiou and M. C. Smith, Topological Approaches to Robustness, *Lecture Notes in Control and Information Sciences*, **185**, pp. 222-241, Springer-Verlag, 1993.
18. T. T. Georgiou and M. C. Smith, "Metric Uncertainty and Nonlinear Feedback Stabilization," in *Feedback Control, Nonlinear Systems, and Complexity*, B.A. Francis and A.R. Tannenbaum (Editors), Springer-Verlag, Lecture Notes in Control and Information Sciences, **202**, pp. 88-98, 1995.



## Refereed Conference Publications

19. T.T. Georgiou, C. Shankwitz and M.C. Smith, "Identification of linear systems: a stochastic approach based on the graph," Proceedings of the 1992 American Control Conference, Chicago, June 1992, pp. 307-312.
20. S. Buddie, T.T. Georgiou, U. Ozguner and M.C. Smith, Flexible structure experiments at JPL and WPAFB, Proceedings of the 1992 American Control Conference, Chicago, June 1992, pp. 1675-1680.
21. C.Chang and T.T. Georgiou, Spectral factorization and Nevanlinna-Pick interpolation: State-space formulae, Proceedings of the 1992 Conference on Decision and Control, Tucson, Arizona, pp. 437-442, December 1992.
22. J.C. Doyle, T.T. Georgiou and M.C. Smith, The parallel projection operators of a non-linear feedback system, Proceedings of the 1992 Conference on Decision and Control, Tucson, Arizona, December 1992, pp. 1050-1054.
23. T.T. Georgiou and M.C. Smith, "Graphs, causality and stabilizability: linear, shift-invariant systems on  $L_2[0, \infty)$ ," Proceedings of the 1992 Conference on Decision and Control, Tucson, Arizona, December 1992, pp. 1024-1029.
24. H. Dym, T.T. Georgiou and M.C. Smith, Explicit formulas for optimally robust controllers for delay systems, Systems and Networks: Mathematical Theory and Applications, Proceedings of the MTNS-93, Regensburg, August 2-6, 1993, Math. Research, vol. 79, Akademie Verlag, pp. 663-666.
25. I. Fialho and T.T. Georgiou "On stability and performance of sampled-data systems subject to wordlength constraint" Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 309-14.
26. C.Chang and T.T. Georgiou, "A note on the Nevanlinna-Schur algorithm with real coefficients," Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 2181-2.
27. H. Dym, T.T. Georgiou and M.C. Smith, "Direct design of optimal controllers for delay systems," Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 3821-3.
28. T.T. Georgiou, "Differential stability and robust control of nonlinear systems," Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 984-9.
29. T.T. Georgiou, "Remarks on Differential stability of nonlinear systems," Proceedings of the 1994 Conference on Decision and Control, pp. 646-647, December 1994.

30. T.T. Georgiou and M.C. Smith, "Intrinsic difficulties in using the doubly-infinite time axis for input-output systems theory," Proceedings of the 1994 Conference on Decision and Control, pp. 2654-2655, December 1994.
31. I. Fialho and T.T. Georgiou, "On the  $\mathcal{L}_1$ -norm of uncertain linear systems, Proceedings of the 1995 American Control Conference, pp. 939-943, June 1995.
32. I. Fialho and T.T. Georgiou, " $\mathcal{L}_1$  state-feedback control with a prescribed rate of exponential convergence," Proceedings of the 1995 American Control Conference, pp. 956-960, June 1995.
33. T.T. Georgiou and M.C. Smith, "Distance Measures for Uncertain Nonlinear Systems," Proceedings of the 1995 European Control Conference, to appear.
34. I. Fialho and T.T. Georgiou, "A variational approach to  $L_\infty$  performance of nonlinear systems" Proceedings of the 1995 Conference on Decision and Control, to appear.
35. T.T. Georgiou and M.C. Smith, "Remarks on robustness of nonlinear systems," Proceedings of the 1995 Conference on Decision and Control, to appear.

## 2 Accomplished Research

### 2.1 Control/modeling with uncertainty in the gap metric

A basic requirement in control system synthesis is that the feedback controller tolerates modeling errors. Thus, a level of robustness to unmodeled dynamics is a prerequisite to any design methodology. A suitable measure for quantifying modeling uncertainty is the so-called gap metric. This metric measures the "aperture" between graphs of dynamical systems. In simple terms, it represents a measure of the distance between possible responses of two given dynamical systems when both operate within the same stabilizing feedback loop. The usefulness of the gap metric stems from the fact that it gives the "coarsest" description of possible perturbations that a feedback system can tolerate (i.e., it induces the coarsest topology under which stability is a robust property). This fact has largely motivated the present research, and our goal has been to develop analysis and synthesis tools which would allow synthesis of controllers with required tolerance to gap uncertainty, as well as a compatible approximation and identification framework.

Abstractly, the graph of a dynamical system  $P$  is the collection of all possible pairs of inputs and outputs which are compatible with the specified dynamics, i.e., the graph of  $P$  is

$$\mathcal{G}_P = \left\{ \begin{pmatrix} u \\ y \end{pmatrix} : y = Pu \text{ where } u, y \text{ are bounded input and output time functions} \right\}.$$

The gap between systems  $P_1$  and  $P_2$  is defined as  $\delta(P_1, P_2) := \max\{\tilde{\delta}(P_1, P_2), \tilde{\delta}(P_2, P_1)\}$ , where

$$\tilde{\delta}(P_1, P_2) = \inf_{w_1 \in \mathcal{G}_{P_1}} \sup_{w_2 \in \mathcal{G}_{P_2}, \|w_2\|=1} \|w_1 - w_2\|.$$

Thus, the gap captures the mismatch between two systems, and represents a quantitative measure of how different their responses might be. The "directed gap" can be computed as follows:

$$\tilde{\delta}(P, P_1) = \inf_{Q \in H_\infty} \left\| \begin{pmatrix} N \\ M \end{pmatrix} - \begin{pmatrix} N_1 \\ M_1 \end{pmatrix} Q \right\|_\infty,$$

and  $P = NM^{-1}$  and  $P_1 = N_1M_1^{-1}$  are normalized right coprime factorizations of the system transfer functions (see [16]).

#### 2.1.1 Optimally robust control system design.

Research reported in [1] demonstrated the use of the gap metric as a design tool and applied it to controller analysis/design for flexible structures at JPL and Wright Patterson AFB.

The basic methodology revolves around an optimization problem which was cast as an  $H_\infty$  optimal control problem.

In the context of linear systems, given a nominal model with transfer function  $P(s)$ , the quantity

$$b(P, C) := \left\| \begin{pmatrix} I \\ P \end{pmatrix} (I - CP)^{-1} (I, -C) \right\|_\infty^{-1},$$

(where “ $I$ ” denotes the identity) represents the maximal level of gap uncertainty that the feedback system with plant  $P$  and controller  $C$  can tolerate (see figure 1). By suitably

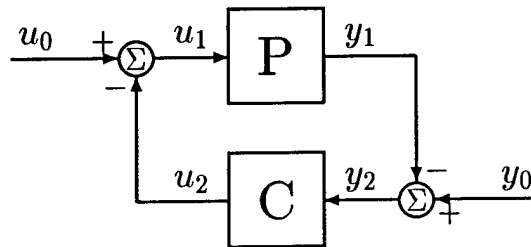


Figure 1: Standard feedback configuration  $[P, C]$ .

scaling inputs and outputs with weight-functions  $W_o$  and  $W_i$ , one is led to the more general robustness radius

$$b(P, C, W_o, W_i) := \left\| \begin{pmatrix} W_i^{-1} \\ W_o P \end{pmatrix} (I - CP)^{-1} (W_i, -CW_o^{-1}) \right\|_\infty^{-1}.$$

The problem of maximizing such a quantity by a proper choice of  $C$  is as a standard ( $H_\infty$ -minimization) problem for which code is now available (e.g., in Matlab, in the  $\mu$ -tools Toolbox). This problem, largely due to its origin in optimization of a gap-robustness radius, is well-posed and leads to a versatile and simple design methodology. The choice of the weights  $W_o$ ,  $W_i$  can be motivated by two alternative ways which are discussed in [1] (one of them is the basis of the so-called “Glover-McFarlane loopshaping” and the other is motivated by the interpretation of the  $b(\cdot, \cdot)$  as a robustness measure).

### 2.1.2 Distributed parameter plants.

The X-29 aircraft at its most unstable flight condition can be represented by a model of the following type:

$$P(s) = \frac{e^{-hs}}{\sigma s - 1}$$

where the time delay  $h$  and the unstable pole location  $\frac{1}{\sigma}$  have a product of about  $h/\sigma \equiv 0.37$  (e.g., see [10]). Such a model is quite challenging to control and has motivated the research undertaken in distributed parameter plants. The research focused to such systems with a time-delay. Due to the  $\infty$ -dimensional nature of the system model, finite-dimensional techniques of standard  $H_\infty$ -optimization theory are not directly applicable.

We considered the problem of assessing how difficult it is to control such a system by computing the largest gap-robustness radius that any controller can stabilize about the nominal model. The solution, for the case of first order systems (such as the one above for the X-29 aircraft) is documented in [21], and for the general higher order case is documented in [8]. In addition, in these references we obtained for the first time a *closed form* expression for the optimally robust controller in the gap metric, i.e., the controller for which  $b(P, C)$  assumes the largest possible value for the given plant  $P$ . Our approach is based on a synthesis of state-space techniques with a certain algebra of "distributed-delay" operators. The form of the transfer function of the optimal controller (see [8, formula (51)]) for general high order plants is:

$$C_{\text{opt}}(s) = \frac{G_1(s)}{G_2(s) - G_3(s)H(h, s)}$$

where  $G_i(s)$  ( $i = 1, 2, 3$ ) are rational transfer function computable from the problem data using finite-dimensional arithmetic, i.e., using linear algebra and information on a realization for the finite-dimensional part of the system model. The expression  $H(h, s)$  represents the transfer function of a distributed parameter delay, i.e., an element which has a finite-duration impulse response. We developed approximation techniques for this type of systems in [22]. Such techniques allow the possibility of a finite-dimensional implementation of a controller with satisfactory levels of robustness and performance. The example  $P(s) = \frac{e^{-hs}}{\sigma s - 1}$ , which coincides with the aforementioned model for the X-29 aircraft, is worked out in detail, and for this model, first and third order approximants to the optimal controller have been generated. Both controllers display a type of phase compensation which is inherent in the gap-optimal controller as well. The basic technique was applied to a system with a resonance as well as a substantial time-delay. For this system it was demonstrated that the technique has certain advantages over the traditional use of Smith predictors for such systems, which are especially common in process control.

In parallel, research reported in [20] was carried out to elucidate certain basic issues which are relevant to control of distributed parameter systems. In particular, well posedness of feedback stability and the tolerance to high frequency uncertainty is studied in detail. The research focused on several physically motivated properties which mathematical models should possess. A system theoretic framework for representation and control of infinite-dimensional systems with finite-dimensional input and output spaces was developed from first principles. Certain issues on the relevance of the two-sided time axis in feedback theory are discussed in [23].

### 2.1.3 Approximation and Model Identification in the Gap metric

Given a nominal model it is often necessary to obtain a lower order approximant that retains some of the basic characteristics. This is invariably dictated by cost and complexity constraints in designing suitable controllers. The research undertaken and reported in [19] led to linking the gap-approximation problem with the framework of Hankel-norm approximation theory. More specifically, if  $P(s)$  is the transfer function matrix of a nominal model of McMillan degree  $n$ , and  $\hat{P}(s)$  denotes a transfer function of a candidate approximant to  $P(s)$  of McMillan degree  $k$ , the following bounds on  $\inf_{\hat{P}} \delta(P, \hat{P})$  hold:

$$\sum_{i=k+1}^n \sigma_i(\Pi_- G^*|_{H_2}) \geq \inf\{\delta(P, \hat{P}) : \hat{P} \text{ of degree } k < n\} \geq \sigma_{k+1}(\Pi_- G G^*|_{H_2^{p+m}}),$$

where  $\sigma_k(\cdot)$  denotes the  $k$ -th singular value of a corresponding Hankel operator  $\Pi_-(\cdot)|_{H_2}$ , and  $G = \begin{pmatrix} M \\ N \end{pmatrix}$  is a graph symbol for  $P(s)$  with  $P(s) = NM^{-1}$  a normalized coprime factorization of  $P(s)$ . The above formula motivates and at the same time highlights the limitations of successive optimal Hankel norm approximation of the symbol  $G$  by lower order symbols, as a technique for obtaining suboptimal approximants. All functions required in this methodology are now available in commercial control system design toolboxes (e.g., the  $\mu$ -tools Toolbox of Matlab). This work is documented in [19].

Research reported in [26] developed a methodology for model identification which is compatible with the robust design framework. It focused on the problem of identifying a nominal model using raw observations of inputs  $u(\ell)$  and outputs  $y(\ell)$  of a linear system ( $\ell = 0, 1, 2, \dots$  in discrete-time). The identification can take place within a closed feedback loop under normal operating conditions by probing the inputs and outputs of the plant. Inputs and outputs are correlated to produce a power spectrum matrix  $f_{x,x}(\theta)$  corresponding to the covariance lags

$$E\left\{\begin{pmatrix} u(\ell) \\ y(\ell) \end{pmatrix} \begin{pmatrix} u(\ell+k) \\ y(\ell+k) \end{pmatrix}^T\right\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ik\theta} f_{x,x}(\theta) d\theta.$$

Spectral factorization of the power spectrum matrix  $f_{x,x}(\theta)$  into

$$f_{x,x}(\theta) = G(e^{i\theta})G^*(e^{i\theta})$$

provides a possible candidate for a graph symbol for the system to be identified. However, due to the variability of the estimated spectrum and because of measurement noise, the spectral factors  $G(e^{i\theta})$  may be square with full rank (instead of having rank = size of input vectors). Approximation techniques are presented which generate admissible graph symbols by suitably approximating  $f_{x,x}(\theta)$  by a power spectrum of appropriate rank (e.g., rank = 1 in the case of SISO systems).  $\mathcal{L}_\infty$ -error bounds are obtained by careful evaluation of the approximation steps. These bounds are then translated into bounds on the gap between the actual plant and the nominal model. Documentation of this work is given in [26], [27].

## 2.2 Modeling uncertainty and robust control for nonlinear systems

Nonlinear control system design presents a number of challenging questions such as: what is the nature of a suitable topology to capture the type of perturbations that stable feedback loops tolerate? how much uncertainty can feedback loops tolerate? and accordingly, what is a suitable measure of robustness for general nonlinear feedback systems? The research under the present grant has given answers to the above questions and has led to an input-output framework for robust control analysis of nonlinear systems. Future work should focus on design issues and case studies.

In [17], [24], [25] a variety of distance measures have been presented for assessing distances between nonlinear systems in different contexts (e.g., when finite-energy or finite-magnitude signals are used, and when these are considered to assume any value or, are known to be bounded with a known a-priori bound). Quantitative margins which guarantee stability of a feedback loop have also been obtained. The computation of the robustness margins amounts to computing an induced norm of a nonlinear operator. For that purpose a methodology has been developed in [14] based on Hamilton-Jacobi theory and tailored to computing norms on bounded signal spaces ( $L_\infty$ -induced norms).

### 2.2.1 Nonlinear gap metrics and robustness guarantees

In [17], [24], [25] several alternative generalizations of the gap metric for nonlinear systems were introduced which permit robustness analysis of nonlinear feedback loops; e.g., the feedback loop in Figure 1 with  $P$ ,  $C$  being general nonlinear systems. These distance measures apply in a variety of situations and depend on the norm chosen for the spaces of signals, as well as, possibly, on a known range of values that the signals are allowed to assume. Accordingly, suitable robustness margins have been obtained.

In all cases, the robustness margin relates to the “minimal opening” between the graph of the plant and the controller and can be expressed as follows:

$$b(P, C) := \|\Pi_{\mathcal{G}_P // \mathcal{G}_C}\|^{-1}.$$

The expression reduces to the one given in the linear case, and  $\Pi_{\mathcal{G}_P // \mathcal{G}_C}$  represents a nonlinear projection onto the graph of  $P$  parallel to the graph of  $C$ . This object was introduced and studied in [7]. The norm varies according to the type of stability one is interested. Typically, the indicated norm is either an induced norm, an incremental gain, or a induced norm computed over a set of bounded signals, depending on the application at hand.

In general, the computation of the robustness margin amounts to computing a certain induced norm of a nonlinear operator. Subsequent research under the present grant focused on computing  $\mathcal{L}_\infty$ -induced norms (see below).

### 2.2.2 $\mathcal{L}_\infty$ -gain analysis of nonlinear systems

A framework for  $\mathcal{L}_\infty$ -gain analysis of general nonlinear systems was developed in [14] [15]. The problem data are in the form of standard nonlinear state equations:

$$\begin{aligned}\dot{x}(t) &= f(t, x(t), u(t)) \\ y(t) &= h(t, x(t), u(t)),\end{aligned}$$

with  $t \in [0, T]$ ,  $x$  the state vector, and  $u, y$  the input and output vectors respectively. If  $\Sigma$  denotes the above system, the induced  $\mathcal{L}_\infty$ -norm over sets of bounded signals (bounded by  $\alpha$ ) is given as follows:

$$\|\Sigma\|_\alpha := \sup_{\|u\|_\infty \leq \alpha} \frac{\|\Sigma u\|_\infty}{\|u\|_\infty} = \sup_{a \in (0, \alpha]} \frac{V_a(t, 0)}{a},$$

where

$$V_a(t, x) = \sup_{\|u\|_\infty \leq \alpha, x(t)=x} \left\{ \text{ess. sup}_{t \leq \tau \leq T} |h(\tau, x(\tau), u(\tau))| \right\}. \quad (1)$$

It was shown in [14] that in the finite horizon case (i.e., when  $T$  is finite),  $V$  is the unique continuous viscosity solution of the Bellman equation

$$\max \left\{ \max_{|u| \leq a} |h(t, x, u)| - v(t, x), \frac{\partial v}{\partial t}(t, x) + \max_{|u| \leq a} \frac{\partial v}{\partial x}(t, x) \cdot f(t, x, u) \right\} = 0, \quad (2)$$

and, in the infinite horizon case (i.e., when  $T = \infty$ ), it is the minimal lower semicontinuous viscosity solution of

$$\max \left\{ |h(x)| - v(x), \max_{|u| \leq a} \frac{\partial v}{\partial x}(x) \cdot f(x, u) \right\} = 0. \quad (3)$$

Comparison theory was developed for the above variational inequalities, and approximation schemes were presented in the framework of discrete dynamic programming. The theory was illustrated on typical cases of saturated feedback loop such as the one shown in Figure 2 (where  $\text{sat}(\cdot)$  denotes a standard saturation nonlinearity to a value equal to one in magnitude). A representative plot of  $V_\alpha$ , for  $\alpha = 1$ , as a function of the 2-vector of states  $x$  is given in Figure 3. The computation of the induced norm requires evaluation of  $V_\alpha$  over a range of values. (E.g., in this case, values  $\alpha \in (0, 1]$  are needed to obtain  $\|\Sigma\|_1 = 1.73$ .) Techniques for reducing the computational requirements are introduced in [14]. Reference [14] documents the findings and conclusions of this research.



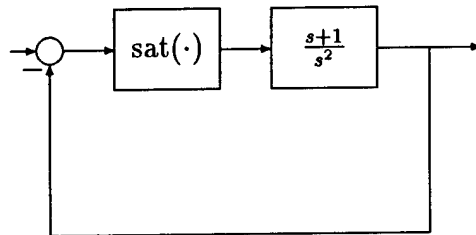


Figure 2: Saturated feedback loop

### 2.3 Linear systems with persistent excitation

Research on this topic addressed problems of analysis and control system design for linear, discrete-time, uncertain systems described in state-space. The values for the state-space realization data belong to a known set of finite possibilities. This effort follows in a direction influenced by a number of authors (Dahleh, Shamma, Pearson, Blanchini, Gutman), on systems affected by persistent excitations. The research led to an algorithm for computing the  $\ell_\infty$ -induced norm [12]. This algorithm is based on ideas from convex analysis and leads naturally to vertex type of results in the  $\ell_\infty$ -induced framework. The work reported in [13] dealt with possible tradeoffs between forced and unforced behavior in state-feedback design for  $\ell_\infty$ -induce norm minimization. In particular, in [13], general necessary and sufficient conditions were obtained for the existence of static nonlinear controllers that achieve a prespecified performance and, at the same time, allow exponentially stable closed loop with a prespecified rate of convergence. An algorithm was given for computing an upper-bound for the achievable performance and it was shown that a variable structure linear controller can achieve performance arbitrarily close to this upper bound. Finally sufficient conditions were given under which this upper bound equals the optimal performance.

### 2.4 Sampled-data systems: performance vs. robustness tradeoffs due to finite wordlength

Research on this topic, reported in [11], focused on implementation issues of sampled-data controllers and, in particular, the effects of finite wordlength in achievable performance. It was shown in [11] that a basic tradeoff exists between performance and robustness, and that

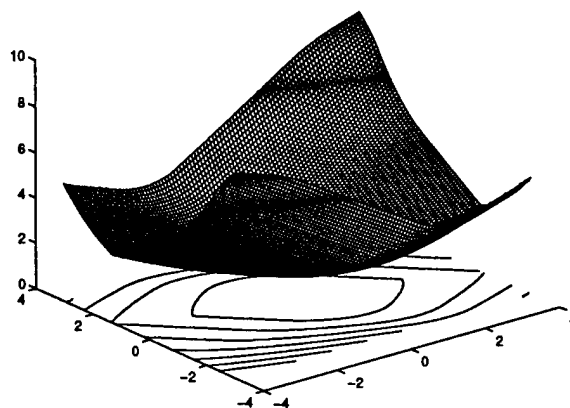


Figure 3:  $V_{\alpha=1}(x)$

this fact, may serve to dictate the selection of sample rate or the wordlength required in a given implementation.

The concept of statistical wordlength, borrowed from the signal processing literature, was used to quantify uncertainty due to coefficient quantization in a digital controller implementation. The following two questions of practical interest were addressed: (a) given a specific controller implementation and a sampling rate, determine the wordlength needed to ensure stability/performance of a closed loop sampled-data system, and (b) given a fixed wordlength and sampling rate, design a sampled-data control to achieve a required level of  $L_2$ -performance.

The stability question in (a) is formulated as a problem of determining the stability radius to structured perturbation. Directional derivatives of an equivalent lifted system are used to estimate the effect of coefficient quantization on input-output performance. The wordlength required to ensure stability and performance, with a prespecified probability margin (e.g., 97.77%), is given by a suitable function of the number of quantized coefficients, the value of a structured stability radius of the problem, and estimates of the sensitivity of certain input-output to uncertainty. A representative plot, reproduced from [11], is given in Figure 4 and shows the wordlength  $W$  required, of a specified controller implementation, for a given level of performance as a function of the sampling period  $h$ . Such plots can be used to point to a judicious choice of sampling period. This tool is especially useful in applications where fast fixed arithmetic processors are used in control applications. A design example was pointed out to demonstrate that an optimal design, under infinite precision arithmetic, may not necessarily be optimal when finite-precision is taken into account. This fact motivated the question in (b) which was cast as a constrained optimization problem. Documentation is provided in [11].

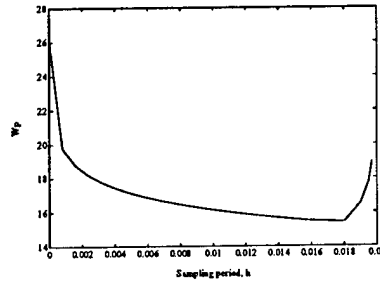


Figure 4:  $W$  vs.  $h$

## 2.5 Spectral factorization: algorithmic aspects

Research on this topic focused on developing an approach to spectral factorization of matrix-valued function which is based on interpolation theory. An algorithm was developed to operate directly on state-space data, and various schemes for fast convergence were proposed. The framework encompasses as a special case the so-called “fast filtering/Chandrashekar” type of algorithms. The approach provides an alternative to existing popular algorithms based on the Riccati equation. The research is documented in [2], [3], [5], [6].

## References

- [1] S.Buddie, T.T. Georgiou, Ümit Özgüner, and M.C. Smith, "Flexible Structure Experiments at JPL and WPAFB", *International Journal of Control*, **58(1)**: 1-19, 1993.
- [2] C. Chang, *Spectral Factorization using Interpolation Theory*, Ph.D. thesis, University of Minnesota, 1993.
- [3] C.Chang and T.T. Georgiou, Spectral factorization and Nevanlinna-Pick interpolation: State-space formulae, Proceedings of the 1992 Conference on Decision and Control, Tucson, Arizona, pp. 437-442, December 1992.
- [4] C.Chang and T.T. Georgiou, "Geometric aspects of the Caratheodory extension problem," *Linear Algebra and its Applications*, **203/4**: 209-251, 1994.
- [5] C.Chang and T.T. Georgiou, "A note on the Nevanlinna-Schur algorithm with real coefficients," Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 2181-2.
- [6] C.Chang and T.T. Georgiou, "Spectral factorization and Nevanlinna-Pick interpolation: State-space formulae", preprint January 1993; *IEEE Trans. on Circuits and Systems*, under revision.
- [7] J.C. Doyle, T.T. Georgiou, and M.C. Smith, "The parallel projection operators of a nonlinear feedback system," *Systems and Control letters*, **20**: 79-85, 1993.
- [8] H. Dym, T.T. Georgiou, and M.C. Smith, "Explicit formulas for optimally robust controllers for delay systems," *IEEE Trans. on Automatic Contr.* , **40(4)**: 656-669, April 1995.
- [9] H. Dym, T.T. Georgiou and M.C. Smith, Direct design of optimal controllers for delay systems, Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 3821-3.
- [10] D. Enns, H. Özbay, and A. Tannenbaum, "Abstract model and controller design for an unstable aircraft," *Journal of Guidance, Control, and Dynamics*, **15(2)**: 498-508, 1992.
- [11] I. Fialho and T.T.Georgiou "On Stability and performance of sampled-data systems subject to wordlength constraint" *IEEE Trans. on Automatic Contr.* , **39(12)**: 2476-2481, December 1994, Also in the Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 309-14.
- [12] I. Fialho and T.T.Georgiou "On the  $\mathcal{L}_1$  norm of uncertain linear systems," to be submitted for publication to *IEEE Trans. on Automatic Control*.

- [13] I. Fialho and T.T. Georgiou, " $\ell_1$  state-feedback control with a prescribed rate of exponential convergence," *IEEE Trans. on Automatic Contr.* , submitted for publication, May 1995.
- [14] I. Fialho and T.T. Georgiou, " $\mathcal{L}_\infty$ -gain analysis of nonlinear systems," *IEEE Trans. on Automatic Contr.* , submitted for publication, May 1995.
- [15] I. Fialho, *Analysis and Synthesis of Nonlinear Feedback Systems over  $L_\infty$* , Ph.D. thesis, Center for Control and Dynamical Systems, University of Minnesota, to be completed Dec. 1995.
- [16] T.T. Georgiou, "On the computation of the gap metric", *Systems and Control Letters*, 11(4): 253-257, October 1988.
- [17] T.T. Georgiou, "Differential stability and robust control of nonlinear systems," *Mathematics of Control, Signals and Systems*, 6(4): 289-307, 1993. Also in Proceedings of the 1993 Conference on Decision and Control, San Antonio, Texas, December 1993, pp. 984-9.
- [18] T.T. Georgiou, "Remarks on Differential stability of nonlinear systems," submitted to the 1994 Conference on Decision and Control, February 1994.
- [19] T.T. Georgiou and M.C. Smith, "Approximation in the gap metric: upper and lower bounds" *IEEE Trans. on Automatic Control*, 38(6): 946-951, 1993.
- [20] T.T. Georgiou and M.C. Smith, "Graphs, causality and stabilizability: linear, shift-invariant systems on  $L_2[0, \infty)$ ," *Mathematics of Control, Signals and Systems*, 6(3): 195-223, 1993.
- [21] T.T. Georgiou and M.C. Smith, "Robust stabilization in the gap metric: Controller design for distributed plants", *IEEE Trans. on Automatic Control*, 37: 1133-1143, August 1992.
- [22] T. T. Georgiou and M. C. Smith, Topological Approaches to Robustness, *Lecture Notes in Control and Information Sciences*, 185, pp. 222-241, Springer-Verlag, 1993.
- [23] T.T. Georgiou and M.C. Smith, "Intrinsic difficulties in using the doubly-infinite time axis for input-output systems theory," *IEEE Trans. on Automatic Contr.* , 40(3): 516-518, March 1995.
- [24] T. T. Georgiou and M. C. Smith, "Metric Uncertainty and Nonlinear Feedback Stabilization," in *Feedback Control, Nonlinear Systems, and Complexity*, B.A. Francis and A.R. Tannenbaum (Editors), Springer-Verlag, Lecture Notes in Control and Information Sciences, 202, pp. 88-98, 1995.

- [25] T.T. Georgiou and M.C. Smith, "Distance Measures for Uncertain Nonlinear Systems," Proceedings of the 1995 European Control Conference, to appear. Journal submission in preparation.
- [26] T.T. Georgiou, C. Shankwitz and M.C. Smith, "Identification of linear systems: a stochastic approach based on the graph," Proceedings of the 1992 ACC, June 1992, pp. 307-312.
- [27] C. Shankwitz, *Identification of Dynamical Systems*, Ph.D. thesis, University of Minnesota, June 1992.
- [28] C. Shankwitz and T.T. Georgiou, "On the envelope of spectral power of stochastic processes," *IEEE Trans. on Information Theory*, **40(2)**: 584-588, 1994.

### **3 Applications to Engineering**

#### **3.1 Robust control system design: uncertainty measured in the gap metric**

The use of the results obtained in the present research have been highlighted in design studies (at JPL and WPAFB; see [1]) and shown to allow easy and effective procedures for the design of control systems. Some of the key algorithmic routines of the methodology are now incorporated in a commercial computer software package (cf. "gap, ncfsyn" commands in Matlab's  $\mu$ -tools toolbox).

#### **3.2 Robust control of nonlinear systems**

Robustness analysis tools in the form of computational algorithms for induced-norms and robustness margins have been developed. Documentation is given in [14] and code will be included in the Ph.D. thesis [15]. It is expected that in subsequent phase these algorithms will form a part of a nonlinear control toolbox.

#### **3.3 Sampled-data systems**

The research demonstrated a basic tradeoff between robustness and performance in the presence of finite wordlength implementation of sampled-data controllers. A statistical wordlength was introduced as a measure of the required wordlength for a given performance level. This research is expected to be important in those engineering applications where very fast sampling is required, thereby enforcing hard constraints on the wordlength available to meet the required processing speed. These results have not been disseminated yet.

#### **3.4 Spectral factorization**

Computer code, written in Matlab, for spectral factorization of matrix-valued functions is available and documented in [2].

## 4 Personnel

Faculty partially supported :	Tryphon T. Georgiou (P.I.)
Students partially supported:	Chin Chang (Ph.D., graduated, 1993)
	Ian Fialho (expected to graduate Dec. 1995)
	Rick Lind
	Rosamond Dolid
	Mayank Amin

## 5 Ph.D. theses

- 1) Chin Chang, *Spectral Factorization using Interpolation Theory*, Ph.D. thesis, Center for Control and Dynamical Systems, University of Minnesota, 1993.
- 2) Ian Fialho, *Analysis and Synthesis of Nonlinear Feedback Systems over  $L_\infty$* , Ph.D. thesis, Center for Control and Dynamical Systems, University of Minnesota, to be completed in December 1995.

## 6 Awards

*1992 IEEE Control Systems Society G.S. Axelby Outstanding Paper Award.*  
Awarded to the P.I. and his co-author, for the paper:  
T.T. Georgiou and M.C. Smith "Optimal robustness in the gap metric",  
*IEEE Trans. on Automat. Contr.*, vol. 35(6), 673-686.